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Tutorial 7

2.

Given a k-CNF formula, if k <= 3, then no conversion takes place and the solution is found. When k >= 3, let the clauses in the CNF formula ø be c1, c2, …cs, and the total number of clauses which size is more than 3 be s(ø). There exists CNF formulas ø0, ø1, ø2, ø3, …. such that s(ø0) > s(ø1), and s(ø1) > s(ø2)….. Since s(ø0) is decreasing over time, there exists a s(øk) where it satisfies s(øk) = 0, which in other words, is the required 3-CNF formula.

Let the additional set of variables be Y. Initially, since c1, c2, …cs only consists of {x1, x2,…,xn}, Y is just an empty set. At every iteration, we insert an additional variable into Y, and let that variable be yt. Let the assignment of value of {y1, y2,…,yt} at time t be bt, and the assignments of {x1, x2,…,xn} be at. For at and bt to be true, there exist an assignment to the value of yt if and only if at-1 and bt-1 is true. Since yt is the only variable assigned, if at and bt are the truth assignments such that they are true, then all the variables assigned except yt satisfies at-1 and bt-1.

Next, let the clause ch be a clause in c1, c2, …cs which ch has more than 3 variables and of size h. From the resolution algorithm in the reversed from, we can observe that (x1 x2 ……xh) => (x1x2y1)(y1x3….xh). Since the additional variable y1 appears in the former clause and its negation appear in the latter clause, it can be deleted, which is equivalent to the clause before deletion. Notice that the clause after the insertion of y1 has a size of 3 and h – 1. As we keep running the algorithm, we get the following clauses:

(x1x2y1)(y1x3….xh)

=> (x1x2y1)(x3y1  y2)(y2c4…..xh)

=> (x1x2y1)(x3y1  y2)(x4x2  y3)(y3x5…..xh)

….

We can see that at each iteration, any clause with > 3 variables can be reduced to 3, and the clause before reduction is satisfiable if the clauses after reduction is also satisfiable. If we keep applying this to ø0, we will have removed a clause with > 3 variables, and repeatedly applying this to the remaining clauses with > 3 variables. In other words, we will get a clause of size h-2, h-3, h-4, …. and will get the formula with s(øk) = h = 0, which is the desired 3-CNF formula.